Int. J. Heat Mass Transfer. Vol. 15, pp. 2371–2383. Pergamon Press 1972. Printed in Great Britain

# STATISTICAL TRANSFER THEORY IN NON-HOMOGENEOUS TURBULENCE

**B. A. KOLOVANDIN and I. A. VATUTIN**
Heat and Mass Transfer Institute, Byelorussian Academy of Sciences, Minsk, USSR

(*Received* 14 *September* 1970)

**Abstract**—One of the possible models of statistical description of momentum and scalar property (temperature, conservative mass concentration) transfer in a non-homogeneous turbulent incompressible flow is considered. The model is based on the use of finite number of one-point correlation equations. To determine the equations some approximate expressions for anisotropic two-point correlations and relationships characterizing momentum and scalar property transfer in homogeneous and anisotropic turbulence are used.

## NOMENCLATURE

$x_i$, Cartesian coordinates ($i = 1, 2, 3$);
$\tau$, time;
$U_i$, averaged velocity;
$u_i$, velocity fluctuations;
$P$, averaged pressure;
$p$, pressure fluctuations;
$\rho$, density;
$\nu$, kinematic viscosity;
$\lambda$, coefficient of molecular transfer of scalar property;
$\Delta_x, \Delta_\xi$, Laplace operators with respect to $x_i$ and $\xi_i$;
$L$, differential operator with respect to $\xi_i$;
$B_{lm,\dots,p}$, two-point correlation;
$A(\xi_s^2), B(\xi_s^2); C(\xi_s^2)$, scalar coefficients of anisotropic correlations;
$\delta_{ij}$, Kronecker delta;
$\mu_n$, velocity component across the direction between two points;
$\Gamma$, averaged scalar property;
$\gamma$, scalar property fluctuation.

### Subscripts

$A, B$, refer to two points of interest;
0, denotes $\vec{\xi} = 0$.

### Superscripts

$'$, value of function at point B;
$*$, isotropic value;
$\rightarrow$, vector quantity;
(0); (2), appropriate values refer to correlations of scalar property and velocity correlations of the second order, respectively;
$l$, average value.

## 1. MOMENTUM TRANSFER

THE APPROXIMATE statistical description of the momentum transfer in nonhomogeneous turbulence is started with the pair of equations for double and triple correlations†

$$\frac{\partial}{\partial \tau}\overline{u_i u_j} + U_k \frac{\partial}{\partial x_k}\overline{u_i u_j} + \overline{u_i u_k}\frac{\partial U_j}{\partial x_k} + \overline{u_j u_k}\frac{\partial U_i}{\partial x_k} + \frac{\partial}{\partial x_k}\overline{u_i u_j u_k} - \nu \Delta_x \overline{u_i u_j} + 2\nu \overline{\frac{\partial u_i}{\partial x_k}\cdot\frac{\partial u_j}{\partial x_k}} + \frac{1}{\rho}\left(\overline{u_i \frac{\partial p}{\partial x_j}} + \overline{u_j \frac{\partial p}{\partial x_i}}\right) = 0, \quad (1.1)$$

† Here and below summation over the repeated indices is implied.

$$\frac{\partial}{\partial\tau}\overline{u_iu_ju_k} + U_l\frac{\partial}{\partial x_l}\overline{u_iu_ju_k} + \overline{u_iu_ju_l}\frac{\partial U_k}{\partial x_l} + \overline{u_ju_ku_l}\frac{\partial U_i}{\partial x_l} + \overline{u_iu_ku_l}\frac{\partial U_j}{\partial x_l}$$
$$- \left(\overline{u_iu_j}\frac{\partial}{\partial x_l}\overline{u_ku_l} + \overline{u_ju_k}\frac{\partial}{\partial x_l}\overline{u_iu_l} + \overline{u_iu_k} \times \frac{\partial}{\partial x_l}\overline{u_ju_l}\right) + \frac{\partial}{\partial x_l}\overline{u_iu_ju_ku_l} - \nu\Delta_x\overline{u_iu_ju_k}$$
$$+ 2\nu\left(\overline{u_i\frac{\partial u_j}{\partial x_l}\cdot\frac{\partial u_k}{\partial x_l}} + \overline{u_j\frac{\partial u_i}{\partial x_l}\cdot\frac{\partial u_k}{\partial x_l}} + \overline{u_k\frac{\partial u_j}{\partial x_l}\cdot\frac{\partial u_i}{\partial x_l}}\right)$$
$$+ \frac{1}{\rho}\left(\overline{u_iu_j\frac{\partial p}{\partial x_k}} + \overline{u_ju_k\frac{\partial p}{\partial x_i}} + \overline{u_iu_k\frac{\partial p}{\partial x_j}}\right) = 0 \quad (1.2)$$

which are rigorous consequences of the Navier–Stokes and Reynolds equations

$$\frac{\partial U_i}{\partial\tau} + U_k\frac{\partial U_i}{\partial x_k} + \frac{\partial}{\partial x_k}\overline{u_iu_k} + \frac{1}{\rho}\frac{\partial P}{\partial x_i} - \nu\Delta_x U_i = 0, \quad (1.3)$$

and the continuity equation which for the averaged velocity is of the form

$$\frac{\partial U_k}{\partial x_k} = 0. \quad (1.4)$$

It is also assumed that Millionshchikov's hypothesis [1]

$$\overline{u_iu_ju_ku_l} = \overline{u_iu_j}\cdot\overline{u_ku_l} + \overline{u_iu_k}\cdot\overline{u_ju_l} + \overline{u_iu_l}\cdot\overline{u_ju_k} \quad (1.5)$$

is approximately valid. Though expression (1.5) holds strictly only for the random fields with Gaussian probability density distribution, numerous data [2–5] indicate that it may be applied approximately to non-homogeneous turbulent fields as well. This hypothesis allows the set of correlation equations to be restricted by the equation of triple correlations.

For discussion of "unknown" correlations (i.e. those which are not determined by the set of equations under consideration) a new coordinate system

$$\xi_k = (x_k)_B - (x_k)_A, \qquad (x_k)_{AB} = \tfrac{1}{2}[(x_k)_A + (x_k)_B] \quad (1.6)$$

is introduced. These coordinates allow the correlations characterizing the decay of turbulence in equations (1.1) and (1.2) and correlations which include pressure fluctuations to be expressed in the form

$$\overline{\frac{\partial u_i}{\partial x_k}\cdot\frac{\partial u_j}{\partial x_k}} = \tfrac{1}{4}\Delta_x\overline{u_iu_j} - (\Delta_\xi\overline{u_iu_j'})_0; \quad (1.7)$$

$$\overline{u_i\frac{\partial u_j}{\partial x_l}\cdot\frac{\partial u_k}{\partial x_l}} + \overline{u_j\frac{\partial u_i}{\partial x_l}\cdot\frac{\partial u_k}{\partial x_l}} + \overline{u_k\frac{\partial u_i}{\partial x_l}\cdot\frac{\partial u_j}{\partial x_l}}$$
$$= \frac{3}{2}\Delta_x\overline{u_iu_ju_k} - \frac{1}{2}[(\Delta_\xi\overline{u_iu_ju_k'})_0 + (\Delta_\xi\overline{u_iu_ku_j'})_0 + (\Delta_\xi\overline{u_ju_ku_i'})_0]; \quad (1.8)$$

$$\overline{u_i\frac{\partial p}{\partial x_j}} + \overline{u_j\frac{\partial p}{\partial x_i}} = \frac{1}{2}\left(\frac{\partial}{\partial x_j}\overline{pu_i} + \frac{\partial}{\partial x_i}\overline{pu_j}\right) + \left(\frac{\partial}{\partial\xi_j}\overline{u_ip'}\right)_0 + \left(\frac{\partial}{\partial\xi_i}\overline{u_jp}\right)_0; \quad (1.9)$$

$$\overline{u_iu_j\frac{\partial p}{\partial x_k}} + \overline{u_ju_k\frac{\partial p}{\partial x_i}} + \overline{u_iu_k\frac{\partial p}{\partial x_j}} = \frac{1}{2}\left(\frac{\partial}{\partial x_k}\overline{u_iu_jp} + \frac{\partial}{\partial x_i}\overline{u_ju_kp} + \frac{\partial}{\partial x_j}\overline{u_iu_kp}\right) + \left(\frac{\partial}{\partial\xi_k}\overline{u_iu_jp'}\right)_0$$
$$+ \left(\frac{\partial}{\partial\xi_i}\overline{u_ju_kp'}\right)_0 + \left(\frac{\partial}{\partial\xi_k}\overline{u_iu_jp'}\right)_0, \quad (1.10)$$

where the superscript denotes that appropriate fluctuation belongs to point $B$.

Consider the differential equations for the one-point correlations $\overline{pu_r}$ and $\overline{pu_ru_s}$ which are included in (1.9) and (1.10). As is known [6], pressure fluctuations satisfy the Poisson equation

$$\frac{1}{\rho}(\Delta_x)_B p' = -2\left(\frac{\partial}{\partial x_n}\right)_B U_m'\cdot\left(\frac{\partial}{\partial x_m}\right)_B u_n' - \left(\frac{\partial^2}{\partial x_m\partial x_n}\right)_B (u_m'u_n' - \overline{u_m'u_n'}) \quad (1.11)$$

whence the equations for the above correlations may be obtained in the form

$$\frac{1}{\rho}\left[\frac{1}{4}\Delta_x\overline{u_r p} + (\Delta_\xi\overline{u_r p'})_0 + \frac{\partial}{\partial x_k}\left(\frac{\partial}{\partial \xi_k}\overline{u_r p'}\right)_0\right]$$

$$= -2\frac{\partial U_m}{\partial x_n}\cdot\left[\frac{1}{2}\frac{\partial}{\partial x_m}\overline{u_n u_r} + \left(\frac{\partial}{\partial \xi_m}\overline{u_r u_n'}\right)_0\right]$$

$$-\frac{1}{4}\frac{\partial^2}{\partial x_m \partial x_n}\overline{u_m u_n u_r} - \frac{1}{2}\left[\frac{\partial}{\partial x_m}\left(\frac{\partial}{\partial \xi_n}\overline{u_r u_m' u_n'}\right)_0\right.$$

$$\left.+\frac{\partial}{\partial x_n}\left(\frac{\partial}{\partial \xi_m}\overline{u_r u_m' u_n'}\right)_0\right]$$

$$-\left(\frac{\partial^2}{\partial \xi_m \partial \xi_n}\overline{u_r u_m' u_n'}\right)_0; \qquad (1.12)$$

$$\frac{1}{\rho}\left[\frac{1}{4}\Delta_x\overline{u_r u_s p} + (\Delta_\xi\overline{u_r u_s p'})_0 + \frac{\partial}{\partial x_l}\left(\frac{\partial}{\partial \xi_l}\overline{u_r u_s p'}\right)_0\right]$$

$$= -2\frac{\partial U_m}{\partial x_n}\left[\frac{1}{2}\frac{\partial}{\partial x_m}\overline{u_n u_r u_s} + \left(\frac{\partial}{\partial \xi_m}\overline{u_r u_s u_n'}\right)_0\right]$$

$$-\frac{1}{4}\frac{\partial^2}{\partial x_m \partial x_n}\overline{u_m u_n u_r u_s} - \frac{1}{2}\left[\frac{\partial}{\partial x_m}\right.$$

$$\left.\times\left(\frac{\partial}{\partial \xi_n}\overline{u_r u_s u_m' u_n'}\right)_0 + \frac{\partial}{\partial x_n}\left(\frac{\partial}{\partial \xi_m}\overline{u_r u_s u_m' u_n'}\right)_0\right]$$

$$-\left(\frac{\partial^2}{\partial \xi_m \partial \xi_n}\overline{u_r u_s u_m' u_n'}\right)_0, \qquad (1.13)$$

where the subscript "0" denotes that the corresponding differential operators are considered at point $\vec{\xi} = 0$.

Consider subsequently the differential equation for the "unknown" function $(\Delta_\xi\overline{u_i u_j'})_0$ included in (1.7). The derivation of the equation is started with the equation of the two-point double correlations for non-homogeneous turbulence, which in terms of variables (1.6) is as follows

$$\frac{\partial}{\partial \tau}\overline{u_i u_j'} + \overline{u_k u_j'}\left(\frac{\partial U_i}{\partial x_k}\right)_A + \overline{u_i u_k'}\left(\frac{\partial U_j}{\partial x_k}\right)_B$$

$$+\frac{1}{2}[(U_k)_A + (U_k)_B] \times \left(\frac{\partial}{\partial x_k}\right)_{AB}\overline{u_i u_j'}$$

$$+[(U_k)_B - (U_k)_A]\cdot\frac{\partial}{\partial \xi_k}\overline{u_i u_j'} + \frac{1}{2}\left(\frac{\partial}{\partial x_k}\right)_{AB}$$

$$\times(\overline{u_i u_j' u_k'} + \overline{u_i u_k u_j'}) + \frac{\partial}{\partial \xi_k}(\overline{u_i u_j' u_k'} - \overline{u_i u_k u_j'})$$

$$+\frac{1}{2\rho}\left[\left(\frac{\partial}{\partial x_i}\right)_{AB}\overline{p u_j'} + \left(\frac{\partial}{\partial x_j}\right)_{AB}\overline{u_i p'}\right]$$

$$-\frac{1}{\rho}\left(\frac{\partial}{\partial \xi_i}\overline{p u_j'} - \frac{\partial}{\partial \xi_j}\overline{u_i p'}\right) - \tfrac{1}{2}\nu\Delta_x\overline{u_i u_j}$$

$$-2\nu\Delta_\xi\overline{u_i u_j'} = 0.$$

By operating upon the above equation with the Laplace operator and setting $\vec{\xi} = 0$, the following is obtained

$$\frac{\partial}{\partial \tau}(\Delta_\xi\overline{u_i u_j'})_0 + U_k\frac{\partial}{\partial x_k}(\Delta_\xi\overline{u_i u_j'})_0 + (\Delta_\xi\overline{u_k u_j'})_0\frac{\partial U_i}{\partial x_k}$$

$$+(\Delta_\xi\overline{u_i u_k'})_0\frac{\partial U_j}{\partial x_k} + \frac{1}{4}\left(\overline{u_i u_k}\cdot\Delta_x\frac{\partial U_j}{\partial x_k} + \overline{u_j u_k}\cdot\Delta_x\right.$$

$$\left.\times\frac{\partial U_i}{\partial x_k} + \Delta_x U_k\cdot\frac{\partial}{\partial x_k}\overline{u_i u_j}\right) + \left(\frac{\partial}{\partial \xi_l}\overline{u_i u_k'}\right)_0$$

$$\times\frac{\partial^2 U_j}{\partial x_l \partial x_k} - \left(\frac{\partial}{\partial \xi_l}\overline{u_j u_k'}\right)_0\cdot\frac{\partial^2 U_i}{\partial x_l \partial x_k}$$

$$+2\left(\frac{\partial^2}{\partial \xi_l \partial \xi_k}\overline{u_i u_j'}\right)_0\cdot\frac{\partial U_k}{\partial x_l} + \frac{1}{2}\frac{\partial}{\partial x_k}[(\Delta_\xi\overline{u_i u_k u_j'})_0$$

$$-(\Delta_\xi\overline{u_j u_k u_i'})_0] - \left[\Delta_\xi\frac{\partial}{\partial \xi_k}(\overline{u_i u_k u_j'} + \overline{u_j u_k u_i'})\right]$$

$$+\frac{1}{2\rho}\left[\frac{\partial}{\partial x_j}(\Delta_\xi\overline{u_i p'})_0 - \frac{\partial}{\partial x_i}(\Delta_\xi\overline{u_j p'})_0\right]$$

$$+\frac{1}{\rho}\left[\Delta_\xi\left(\frac{\partial}{\partial \xi_j}\overline{u_i p'} + \frac{\partial}{\partial \xi_i}\overline{u_j p'}\right)\right]_0$$

$$-\frac{1}{2}\nu\Delta_x(\Delta_\xi\overline{u_i u_j'})_0 - 2\nu(\Delta_\xi\Delta_\xi\overline{u_i u_j'})_0 = 0. \qquad (1.14)$$

Relationships (1.8)–(1.10) as well as equations (1.11)–(1.13) include a number of unknown terms which are differential operators with respect to the variable $\xi_i$ at point $\vec{\xi} = 0$ of two-point correlations

$$[L_{\xi}^{rs\ldots t} B_{lm\ldots,p}(\vec{x}_{AB}, \vec{\xi}, \tau)]_0. \quad (1.15)$$

If the dimensions of the area $d\Omega$ are sufficiently small as compared to the microscales of the appropriate correlations (these characteristic lengths for anisotropic turbulence will be introduced below), the turbulence in $d\Omega$ may be assumed approximately homogeneous.† So, when considering the above terms the approximate model of quasi-homogeneous turbulence may be used, i.e. we may approximately use the expressions of homogeneous two-point correlation tensors of even and odd ranks near the point $\vec{\xi} = 0$ as well as their properties of invariance at reflection and mutual interchanging of $A$ and $B$ points.

Consider some approximate expressions of two-point anisotropic correlations for incompressible fluid. The basic conditions, at least those which are satisfied with the correlation functions between two adjacent points, are as follows: (i) coincidence with the appropriate one-point correlations when the distance between two points is equal to zero ($\vec{\xi} = 0$), i.e.

$$(\overline{u_i u_j \ldots u'_m})_0 = \overline{u_i u_j \ldots u_m} \quad (1.16)$$

(ii) coincidence of anisotropic correlations with the corresponding isotropic correlations at isotropy, i.e.

$$(\overline{u_i u_j \ldots u'_m})^* = Q_{ij\ldots,m} \quad (1.17)$$

where $Q_{ij\ldots,m}$ is two-point isotropic correlation and superscript * denotes the condition of isotropy; (iii) coincidence of differential operators with respect to $\xi_i$ at point $\vec{\xi} = 0$ and the condition of isotropy of two-point anisotropic correlations with the appropriate differential operators of two-point isotropic correlations, i.e.

$$(L_{np\ldots s}\overline{u_i u_j \ldots u'_m})_0^* = L_{np\ldots s} Q_{ij\ldots,m}. \quad (1.18)$$

Because of homogeneous turbulence, two-point correlations possess properties of invariance

$$\overline{c\gamma'}(\xi) = \overline{c\gamma'}(-\xi) = \overline{c'\gamma}(\xi);$$

$$\overline{cu'_i}(\xi) = -\overline{cu'_i}(-\xi) = \overline{c'u_i}(\xi);$$

$$\overline{u_i u'_j}(\xi) = \overline{u_i u'_j}(-\xi) = \overline{u'_i u_j}(\xi);$$

$$\overline{u_i u_j u'_k}(\xi) = -\overline{u_i u_j u'_k}(-\xi) = \overline{u'_i u'_j u_k}(\xi). \quad (1.19)$$

Consider the double two-point anisotropic correlation in detail. In order to satisfy (1.16) and (1.17) we shall express second-rank correlation tensor in the following form

$$\overline{u_i u'_k} = \tfrac{1}{3}\overline{q^2}[A(\xi_s^2)\,\xi_i\xi_k + B(\xi_s^2)\,R_{ik} + C(\xi_s^2) \times (R_{ip}\xi_p\xi_k + R_{kp}\xi_p\xi_i)],$$

where

$$\overline{q^2} = \overline{u_s u_s}; \qquad R_{ik} = 3\frac{\overline{u_i u_k}}{\overline{q^2}}.$$

Conditions (1.16) and (1.19) being taken into account, we express this correlation near the point $\vec{\xi} = 0$ in the form

$$\overline{u_i u'_k} = \tfrac{1}{3}\overline{q^2}[a_0\cdot\xi_i\xi_k + (1 + \tfrac{1}{2}b_{mn}\xi_m\xi_n + \ldots)R_{ik} + c_0(R_{ip}\xi_p\xi_k + R_{kp}\xi_p\xi_i) + \ldots]. \quad (1.20)$$

By fitting expression (1.20) to incompressibility condition

$$\frac{\partial}{\partial\xi_k}\overline{u_i u'_k} = 0$$

and to conditions of invariance, relationship (1.20) can be rewritten in the form

$$\overline{u_i u'_k} = \tfrac{1}{3}\overline{q^2}\{a_0\xi_i\xi_k + [1 - 2(a_0 + 2c_0)r^2]R_{ik} + c_0(R_{ip}\xi_p\xi_k + R_{kp}\xi_p\xi_i)\} \quad (1.21)$$

where

$$r^2 = \xi_s^2.$$

From the expression of double isotropic correlation for adjacent points [9] the following expression follows

$$(\Delta_{\xi}Q_{i,k})_0 = -15\overline{u^2}\cdot\rho_{i,k}^{(2)*}$$

† The idea was originally suggested by Chou [7].

where

$$\rho_{i,k}^{(2)*} = -\frac{1}{3}\left(\frac{\partial^2 f}{\partial r^2}\right)_0 \cdot \delta_{ik}; \qquad \rho_{s,s}^{(2)*} = \frac{1}{l_g^2};$$

$f = \overline{u_r u_r'}/\overline{u^2}$ is the longitudinal correlation coefficient, $l_g$ is the transverse microscale of turbulence. Thus, by allowing for condition (1.18) we obtain the following definition of the microscale of double velocity correlations in anisotropic turbulence

$$\rho_{i,k}^{(2)} = -\frac{1}{5q^2}\cdot(\Delta_\xi \overline{u_i u_k'})_0. \qquad (1.22)$$

By substituting expression (1.21) into (1.22) it is possible to obtain the following relationships for the scalar coefficients

$$a_0 = \tfrac{1}{2}\rho_{s,s}^{(2)} - 2c_0, \qquad \rho_{\rho,\psi}^{(2)} = \tfrac{1}{3}\rho_{s,s}^{(2)}\cdot R_{\rho\psi}$$
$$+ \tfrac{1}{15}\rho_{s,s}^{(2)}\cdot(1 - 4\bar{C}_0)(R_{\rho\psi} - \delta_{\varphi\psi}), \qquad (1.23)$$

where $\bar{C}_0 = c_0/\rho_{s,s}^{(2)}$ is the dimensionless scalar coefficient. From the second relationship (1.23) at any fixed indices $\varphi$ and $\psi$ an expression for the coefficient $\bar{C}_0$ may be obtained.

Utilizing relations (1.23), we express (1.21) in the form

$$\overline{u_i u_k'} = \tfrac{1}{3}\overline{q^2}[\tfrac{1}{2}\rho_{s,s}^{(2)}(1 - 4\bar{C}_0)\xi_i\xi_k$$
$$+ (1 - \rho_{s,s}^{(2)}\cdot r^2)R_{ik} + \rho_{s,s}^{(2)}\cdot C_0$$
$$\times (R_{ip}\xi_p\xi_k + R_{kp}\xi_p\xi_i) + \ldots]. \qquad (1.24)$$

In addition to the above relations which will be useful for determination of the unknown terms in (1.14), in accordance with the condition (1.18) it is possible to obtain some important relationships which will be applied for determination of unknown terms in the system of equations (1.1), (1.2) and (1.11)–(1.13).

First of all it may be shown that power expansion of the first- and third-rank correlation tensors $\overline{u_i\gamma'}$ and $\overline{u_i u_j u_k'}$ begins with the third powers of $\xi_i$, i.e.

$$\left(\frac{\partial}{\partial\xi_j}\overline{u_i\gamma'}\right)_0 = \left(\frac{\partial}{\partial\xi_l}\overline{u_i u_j u_k'}\right)_0 \equiv 0. \qquad (1.25)$$

Then similarly to (1.9) and according to condition (1.18) the following relationships may be obtained

$$\left(\frac{\partial}{\partial\xi_l}\Delta_\xi\overline{u_l u_s u_s'}\right)_0^* = \left(\frac{\partial}{\partial\xi_l}\Delta_\xi Q_{ls,s}\right)_0;$$
$$(\Delta_\xi\Delta_\xi\overline{u_s u_s'})_0^* = (\Delta_\xi\Delta_\xi Q_{s,s})_0. \qquad (1.26)$$

While considering right hand sides of these relations, the well-known dimensionless coefficient is introduced

$$S^* = \frac{\overline{(\partial u_r/\partial x_r)^3}}{[\overline{(\partial u_r/\partial x_r)^2}]^{\frac{3}{2}}} \qquad (1.27)$$

which is the skewness factor of the probability density distribution of velocity fluctuation derivatives at isotropic turbulence, and another dimensionless statistical coefficient [10]

$$S_v^* = 2v\cdot\frac{\overline{(\partial^2 u_r/\partial x_r^2)^2}}{[\overline{(\partial u_r/\partial x_r)^2}]^{\frac{3}{2}}} = \frac{1}{R_q}\cdot S_v \qquad (1.28)$$

where $R_q = q/v(\rho_{s,s}^{(2)*})^{\frac{1}{2}}$ is the turbulent Reynolds number, and

$$S_v = 2\sqrt{3}\,\frac{\overline{u^2}^{\frac{1}{2}}}{(\rho_{s,s}^{(2)*})^{\frac{1}{2}}}\cdot\frac{\overline{(\partial^2 u_r/\partial x_r^2)^2}}{[\overline{(\partial u_r/\partial x_r)^2}]^{\frac{3}{2}}}.$$

It can be shown that the values in the left-hand side of relations (1.26) may be expressed in terms of the microscales $\rho_{m,n}^{(2)*}$, $S^*$ and $S_v^*$. Indeed, the coefficients (1.27) and (1.28) may be rewritten in the form

$$S^* = -\frac{6\sqrt{15}}{7}\cdot\frac{(\partial/\partial\xi_l\Delta_\xi Q_{ls,s})_0}{(\Delta_\xi Q_{s,s})_0^{\frac{3}{2}}};$$

$$S_v^* = -\frac{6\sqrt{15}}{7}\cdot v\cdot\frac{(\Delta_\xi\Delta_\xi Q_{s,s})_0}{(\Delta_\xi Q_{s,s})_0^{\frac{3}{2}}}.$$

Thus, the above values may be presented in the form

$$\left(\frac{\partial}{\partial\xi_l}\Delta_\xi Q_{ls,s}\right)_0 = -\frac{7}{6\sqrt{15}}\cdot S^*\cdot(\Delta_\xi Q_{s,s})_0^{\frac{3}{2}};$$

$$(\Delta_\xi\Delta_\xi Q_{s,s})_0 = \frac{7}{6\sqrt{15}}\cdot S_v^*\cdot\frac{1}{v}\cdot(\Delta_\xi Q_{s,s})_0^{\frac{3}{2}}.$$

By taking these relations and conditions (1.18) into the account, the following expressions are arrived at for homogeneous turbulence

$$\left(\frac{\partial}{\partial \xi_l}\Delta_\xi \overline{u_l u_s u_s'}\right)_0 = -\frac{7}{6\sqrt{15}}\cdot S\cdot(\Delta_\xi \overline{u_s u_s'})_0^{3/2};$$

$$(\Delta_\xi \Delta_\xi \overline{u_s u_s'})_0 = -\frac{7}{6\sqrt{15}}\cdot S_v\cdot\frac{1}{v}\cdot(\Delta_\xi \overline{u_s u_s'})_0^{3/2}, \quad (1.29)$$

where $S$ and $S_v$ are dimensionless statistical coefficients which characterize a homogeneous anisotropic velocity field.

Besides, the unknown terms $(\Delta_\xi \overline{u_r u_s p'})_0$ of equation (1.13) and $(\Delta_\xi(\partial/\partial\xi_r)\overline{u_l p'})_0$ of (1.14) as well as other terms of the type of (1.15) may be determined for homogeneous turbulence. From (1.13) and equation for $(\partial/\partial\xi_r\, \overline{u_l p'})_0$, derived similarly to (1.12), follows the equation

$$\left.\begin{aligned}\frac{1}{\rho}(\Delta_\xi \overline{u_r u_s p'})_0 &= -\left(\frac{\partial^2}{\partial\xi_m \partial\xi_n}\overline{u_r u_s u_m' u_n'}\right)_0;\\ \frac{1}{\rho}\left(\Delta_\xi \frac{\partial}{\partial \xi_r}\overline{u_l p'}\right)_0 &= -\left(\frac{\partial^3}{\partial\xi_{lia}\partial\xi_m\partial\xi_n}\times \overline{u_l u_m' u_n'}\right)_0.\end{aligned}\right\} \quad (1.30)$$

Thus, with conditions of invariance (1.19), second relation in (1.23), double correlation (1.24) between two adjacent points, equality (1.25) and relationships (1.5), (1.7)–(1.10), (1.29) and (1.30) in view, equations (1.1), (1.2) and (1.12)–(1.14) are obtained in the form

$$\frac{\partial}{\partial\tau}\overline{u_i u_j} + U_k\frac{\partial}{\partial x_k}\overline{u_i u_j} + \overline{u_i u_k}\frac{\partial U_j}{\partial x_k} + \overline{u_j u_k}\frac{\partial U_i}{\partial x_k} + \frac{\partial}{\partial x_k}\overline{u_i u_j u_k} + \frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\overline{u_j p} + \frac{\partial}{\partial x_j}\overline{u_i p}\right)$$
$$- \tfrac{1}{2}\gamma\Delta_x \overline{u_i u_j} + \tfrac{10}{3}\cdot v\cdot \overline{\rho_{s,s}}\,[R_{ij} + \tfrac{1}{5}(1-4\bar{C}_0)\times(R_{ij}-\delta_{ij})] = 0;$$

$$\frac{\partial}{\partial\tau}\overline{u_i u_j u_k} + U_l\frac{\partial}{\partial x_l}\overline{u_i u_j u_k} + \overline{u_i u_j u_l}\cdot\frac{\partial U_k}{\partial x_l} + \overline{u_i u_k u_l}\cdot\frac{\partial U_j}{\partial x_l} + \overline{u_j u_k u_l}\cdot\frac{\partial U_i}{\partial x_l} + \overline{u_k u_l}\cdot\frac{\partial}{\partial x_l}$$
$$\times\, \overline{u_i u_j} + \overline{u_j u_e}\cdot\frac{\partial}{\partial x_l}\overline{u_i u_k} + \overline{u_i u_l}\cdot\frac{\partial}{\partial x_l}\overline{u_j u_k} + \frac{1}{2\rho}\left(\frac{\partial}{\partial x_i}\overline{u_j u_k p} + \frac{\partial}{\partial x_j}\overline{u_i u_k p} + \frac{\partial}{\partial x_k}\overline{u_i u_j p}\right)$$
$$- \tfrac{1}{4}v\Delta_x\overline{u_i u_j u_k} = 0;$$

$$\frac{1}{4\rho}\Delta_x\overline{u_r p} + \frac{\partial U_m}{\partial x_n}\cdot\frac{\partial}{\partial x_m}\overline{u_n u_r} + \frac{1}{4}\frac{\partial^2}{\partial x_m \partial x_n}\times\overline{u_m u_n u_r} = 0;$$

$$\frac{1}{4\rho}\Delta_x\overline{u_r u_s p} + \frac{\partial U_m}{\partial x_n}\cdot\frac{\partial}{\partial x_m}\overline{u_n u_r u_s} + \frac{1}{4}\cdot\frac{\partial^2}{\partial x_m\partial x_n}\times(\overline{u_m u_n}\cdot\overline{u_r u_s} + \overline{u_m u_r}\cdot\overline{u_n u_s} + \overline{u_m u_s}\cdot\overline{u_n u_r}) = 0;$$

$$\frac{\partial}{\partial\tau}\overline{\rho_{s,s}} + U_k\frac{\partial}{\partial x_k}\overline{\rho_{s,s}} - \frac{1}{20}\Delta_x U_k\cdot\frac{\partial}{\partial x_k}\overline{q^2} - \frac{1}{10}\overline{u_s u_k}\cdot\frac{\partial}{\partial x_k}\Delta_x U_s + \frac{2}{15}\cdot\overline{\rho_{s,s}}\cdot\frac{\partial U_k}{\partial x_l}\cdot F_{1kl}\frac{2}{15}$$
$$\times\frac{\partial U_k}{\partial x_l}\cdot\overline{\rho_{s,s}}\cdot F^{ss}_{ln} + \frac{7}{3\sqrt{3}}\overline{\rho_{s,s}}^{3/2}\cdot\left(S + \frac{l}{R_q}\cdot S_v\right) - \frac{1}{2}\cdot v\cdot\Delta_x\overline{\rho_{s,s}} = 0,$$

where

$$F_{1lk} = 2(3-2\bar{C}_0)R_{lk} - (1-4\bar{C}_0)\delta_{lk};$$
$$\overline{\rho_{s,s}} = \overline{q^2}\cdot\rho^{(2)}_{s,s};$$
$$F_{2lk} = 4\bar{C}_0\cdot R_{lk} - (5+4\bar{C}_0)\delta_{lk}.$$

The above system of equations together with (1.3) and (1.4) describes the momentum transfer at non-homogneneous turbulence. It is determined but for three coefficients $S$, $S_v$ and $\bar{C}_0$.†

† If the microscale equations are used in a tensor form, the coefficient $\bar{C}_0$ will be determined by the second relationship in (1.2). For the case, like the authors', when the trace of tensor equation (1.14) is applied, the coefficient $\bar{C}_0$ should be found experimentally from the results of measuring two-point velocity correlations in homogeneous turbulence.

The numerical values of the statistical coefficient $S^*$ may be evaluated by the inequality

$$|\overline{f_1 \cdot f_2}| \leqslant \overline{f^2}^{\frac{1}{2}} \cdot \overline{f_2^2}^{\frac{1}{2}} \qquad (1.31)$$

valid for any randon functions $f_1$ and $f_2$. Application of this inequality to the coefficient $S^*$ yields

$$|S^*| \leqslant (\delta_u^*)^{\frac{1}{2}}$$

where $\delta_u^*$ is the flatness factor of the probability density distribution of velocity derivatives.

The above inequality may be made more exact and written as follows [11]

$$|S^*| \leqslant \frac{2}{\sqrt{21}} \cdot (\delta_u^*)^{\frac{1}{2}} \simeq \frac{2}{\sqrt{7}}.$$

This estimate of the coefficient $S^*$ may, probably, be also used as rough estimate of the coefficient $S$.

To determine the coefficient $S_v^*$, the equation for $(\Delta_\xi Q_{s,s})_0$ which results from the dynamic equation of two-point correlations $Q_{s,s}$ may be applied. The use of the first Kolmogroff similarity hypothesis yields

$$S_v^* = -S^*.$$

In fact, taking Uberoi's experiments [10] into account, we obtain

$$S^* = 0{\cdot}45; \qquad S_v^* = 0{\cdot}55.$$

These estimates of the coefficients $S^*$ and $S_v^*$ may, probably be used as rough estimates of the homogeneous coefficients $S$ and $S_v$.

## 2. SCALAR PROPERTY TRANSFER

The following are the basic equations governing scalar property transfer in non-homogeneous turbulence:

(i) averaged transfer equation in the form of the of the diffusion equation [6]

$$\frac{\partial \Gamma}{\partial \tau} + U_k \frac{\partial \Gamma}{\partial x_k} + \frac{\partial}{\partial x_k} \overline{u_k \gamma} = \lambda \Delta_x \Gamma \qquad (2.1)$$

(ii) transport equation for scalar property reduction fluxes [12]

$$\frac{\partial}{\partial \tau} \overline{u_i \gamma} + U_k \frac{\partial}{\partial x_k} \overline{u_i \gamma} + \overline{u_i u_k} \frac{\partial \Gamma}{\partial x_k} + \overline{u_k \gamma} \frac{\partial U_i}{\partial x_k} + \frac{\partial}{\partial x_k} \overline{u_i u_k \gamma} + \frac{1}{\rho} \overline{\gamma \frac{\partial p}{\partial x_i}} - \nu \overline{\gamma \frac{\partial^2 u_i}{\partial x_k^2}} - \lambda \overline{u_i \frac{\partial^2 \gamma}{\partial x_k^2}} = 0; \qquad (2.2)$$

(iii) transport equation for triple correlations included in (2.2)

$$\frac{\partial}{\partial \tau} \overline{u_i u_k \gamma} + U_l \frac{\partial}{\partial x_l} \overline{u_i u_k \gamma} + \overline{u_i u_l \gamma} \frac{\partial U_k}{\partial x_l} + \overline{u_i u_k u_l} \frac{\partial \Gamma}{\partial x_l} + \overline{u_l u_k \gamma} \frac{\partial U_i}{\partial x_l} + \frac{\partial}{\partial x_l} \overline{u_i u_k u_l \gamma} - (\overline{u_i \gamma} \cdot \frac{\partial}{\partial x_l} \overline{u_k u_l} + \overline{u_i u_k} \cdot \frac{\partial}{\partial x_l} \overline{u_l \gamma} + \overline{u_k \gamma} \cdot \frac{\partial}{\partial x_l} \overline{u_i u_l}) + \frac{1}{\rho} \left( \overline{u_i \gamma \frac{\partial p}{\partial x_k}} + \overline{u_k \gamma \frac{\partial p}{\partial x_i}} \right) - \nu \left( \overline{u_i \gamma \frac{\partial^2 u_k}{\partial x_l^2}} + \overline{u_k \gamma \frac{\partial^2 u_i}{\partial x_l^2}} \right) - \lambda \overline{u_i u_k \frac{\partial^2 \gamma}{\partial x_l^2}} = 0. \qquad (2.3)$$

The chain of correlation equations is broken by means of Millionschikov's hypothesis

$$\overline{u_i u_k u_l \gamma} = \overline{u_i u_k} \cdot \overline{u_l \gamma} + \overline{u_i u_l} \cdot \overline{u_k \gamma} + \overline{u_k u_l} \cdot \overline{u_i \gamma}. \qquad (2.4)$$

In the new system of coordinates (1.6) the correlations which in (2.2) and (2.3) characterize the change in values $\overline{u_i \gamma}$ and $\overline{u_i u_k \gamma}$ due to molecular effects and those which include pressure fluctuations are of the form

$$\lambda \overline{u_i \frac{\partial^2 \gamma}{\partial x_k^2}} + \nu \overline{\gamma \frac{\partial^2 u_i}{\partial x_k^2}} = \frac{1}{4} (\lambda + \nu) \Delta_x \overline{u_i \gamma} + (\lambda + \gamma)(\Delta_\xi \overline{u_i \gamma'})_0 + (\lambda - \nu) \times \frac{\partial}{\partial x_k} \left( \frac{\partial}{\partial \xi_k} \overline{u_i \gamma'} \right)_0; \qquad (2.5)$$

$$\lambda\,\overline{u_i u_k \frac{\partial^2 \gamma}{\partial x_l^2}} + \nu\left(\overline{u_i \gamma \frac{\partial^2 u_k}{\partial x_l^2}} + \overline{u_k \gamma \frac{\partial^2 u_i}{\partial x_l^2}}\right)$$

$$= \frac{1}{4}(\lambda + 2\gamma)\Delta_x \overline{u_i u_k \gamma} + \lambda(\Delta_\xi \overline{u_i u_k \gamma'})_0$$

$$+ \nu[(\Delta_\xi \overline{u_i u_k' \gamma'})_0 + (\Delta_\xi \overline{u_k u_i' \gamma'})_0]; \qquad (2.6)$$

$$\overline{\gamma \frac{\partial p}{\partial x_i}} = \frac{1}{2}\frac{\partial}{\partial x_i}\overline{\gamma p} + \left(\frac{\partial}{\partial \xi_i}\overline{\gamma p'}\right)_0; \qquad (2.7)$$

$$\overline{u_i \gamma \frac{\partial p}{\partial x_k}} + \overline{u_k \gamma \frac{\partial p}{\partial x_i}} = \frac{1}{2}\left(\frac{\partial}{\partial x_k}\overline{u_i \gamma p} + \frac{\partial}{\partial x_i}\overline{u_k \gamma p}\right) + \left(\frac{\partial}{\partial \xi_k}\overline{u_i \gamma p'}\right)_0. \qquad (2.8)$$

On writing differential equations for the one-point correlation $\overline{\gamma p}$ and $\overline{u_i \gamma p}$ included in (2.7) and (2.8) and proceeding from Poisson equation (1.11) the following equations are arrived at

$$\frac{1}{\rho}\left[\frac{1}{4}\overline{\gamma p} + (\Delta_\xi \overline{\gamma p'})_0 + \frac{\partial}{\partial x_k}\left(\frac{\partial}{\partial \xi_k}\overline{\gamma p'}\right)_0\right]$$

$$= -2\frac{\partial U_m}{\partial x_m}\left[\frac{1}{2}\frac{\partial}{\partial x_m}\overline{u_n \gamma} + \left(\frac{\partial}{\partial \xi_m}\overline{\gamma u_n'}\right)_0\right]$$

$$-\frac{1}{4}\frac{\partial^2}{\partial x_m \partial x_n}\overline{u_m u_n \gamma} - \frac{1}{2}\left[\frac{\partial}{\partial x_m}\left(\frac{\partial}{\partial \xi_n}\overline{u_m' u_n' \gamma}\right)_0\right.$$

$$\left. + \frac{\partial}{\partial x_n}\left(\frac{\partial}{\partial \xi_m}\overline{u_m' u_n' \gamma}\right)_0\right] - \left(\frac{\partial^2}{\partial \xi_m \partial \xi_n}\overline{u_m' u_n' \gamma}\right)_0; \qquad (2.9)$$

$$\frac{1}{\rho}\left[\frac{1}{4}\Delta_x \overline{u_i \gamma p} + (\Delta_\xi \overline{u_i \gamma p'})_0 + \frac{\partial}{\partial x_k}\left(\frac{\partial}{\partial \xi_k}\overline{u_i \gamma p'}\right)_0\right]$$

$$= -2\frac{\partial U_m}{\partial x_n}\left[\frac{1}{2}\frac{\partial}{\partial x_m}\overline{u_i u_n \gamma} + \left(\frac{\partial}{\partial \xi_m}\overline{u_i \gamma u_n'}\right)_0\right]$$

$$-\frac{1}{4}\frac{\partial^2}{\partial x_m \partial x_n}\overline{u_m u_n u_i \gamma} - \frac{1}{2}\left[\frac{\partial}{\partial x_m}\left(\frac{\partial}{\partial \xi_n}\overline{u_m' u_n' u_i \gamma}\right)_0\right.$$

$$\left. + \frac{\partial}{\partial x_n}\left(\frac{\partial}{\partial \xi_m}\overline{u_m' u_n' u_i \gamma}\right)_0\right]$$

$$-\left(\frac{\partial^2}{\partial \xi_m \partial \xi_n}\overline{u_m' u_n' u_i \gamma}\right)_0; \qquad (2.10)$$

$$\frac{1}{\rho}\left[\frac{1}{4}\Delta_x\left(\frac{\partial}{\partial \xi_k}\overline{u_i \gamma p'}\right)_0 + \left(\Delta_\xi \frac{\partial}{\partial \xi_k}\overline{u_i \gamma p'}\right)_0 + \frac{\partial}{\partial x_j}\left(\frac{\partial^2}{\partial \xi_j \partial \xi_k}\overline{u_i \gamma p'}\right)_0\right]$$

$$= -2\frac{\partial U_m}{\partial x_n}\left[\frac{1}{2}\frac{\partial}{\partial x_m}\left(\frac{\partial}{\partial \xi_k}\overline{u_i \gamma u_n'}\right)_0 + \left(\frac{\partial^2}{\partial \xi_m \partial \xi_k}\overline{u_i \gamma u_n'}\right)_0\right] - \frac{1}{4}\frac{\partial^2}{\partial x_m \partial x_n}$$

$$\times \left(\frac{\partial}{\partial \xi_k}\overline{u_m' u_n' u_i \gamma}\right)_0 - \frac{1}{2}\left[\frac{\partial}{\partial x_m}\right.$$

$$\times \left(\frac{\partial^2}{\partial \xi_n \partial \xi_k}\overline{u_m' u_n' u_i \gamma}\right)_0$$

$$\left. + \frac{\partial}{\partial x_n}\left(\frac{\partial^2}{\partial \xi_m \partial \xi_k}\overline{u_m' u_n' u_i \gamma}\right)_0\right]$$

$$-\left(\frac{\partial^3}{\partial \xi_m \partial \xi_n \partial \xi_k}\overline{u_m' u_n' u_i \gamma}\right)_0$$

$$-\frac{1}{2}\cdot\frac{\partial^2 U_m}{\partial x_n \partial x_k}\cdot\frac{\partial}{\partial x_m}\overline{u_n u_i \gamma}. \qquad (2.11)$$

For determination of the above equations (2.2), (2.3) and (2.9)–(2.11) consider some relationships which characterize scalar property transfer in homogeneous turbulence. First of all, by using the expression of two-point double correlation of scalar property for adjacent points at isotropic turbulence [5], it may be shown that

$$\left(\frac{\partial^2}{\partial \xi_m \partial \xi_n} Q_{\gamma,\gamma}\right)_0 = -6\cdot\overline{\gamma^2}\cdot\rho_{m,n}^{(0)*}$$

where

$$\rho_{m,n}^{(0)*} = -\frac{1}{6}\left(\frac{\partial^2}{\partial r^2}R_{\gamma,\gamma}\right)_0\cdot\delta_{mn};$$

$$\rho_{\gamma,\gamma}^{(0)*} = -\frac{1}{2}\left(\frac{\partial^2}{\partial r^2}R_{\gamma,\gamma}\right)_0 = -\frac{1}{\lambda_\gamma^2};$$

$R_{\gamma,\gamma} = \overline{\gamma\gamma'}/\overline{\gamma^2}$ is the coefficient of two-point correlation of scalar property, $\lambda_\gamma$ is the microscale of scalar property fluctuations.

Then, condition (1.18) being allowed for, the definition is introduced of the microscale of scalar property fluctuations at homogeneous turbulence

$$\rho^{(0)}_{m,n} = -\frac{1}{6\overline{\gamma^2}}\cdot\left(\frac{\partial^2}{\partial\xi_m\partial\xi_n}\overline{\gamma\gamma'}\right)_0;$$

$$\rho^{(0)}_{\gamma,\gamma} = -\frac{1}{6\overline{\gamma^2}}\cdot(\Delta_\xi\overline{\gamma\gamma'})_0. \qquad (2.12)$$

It may also be shown that the following relationships hold for isotropy

$$(\Delta_\xi Q_{ik,\gamma})_0 = -6(\sqrt{2})\cdot S_1^*\cdot\overline{u^2}\cdot[\overline{\gamma^2}]^{\frac{1}{2}} \times\left(\frac{\rho^{(0)*}_{\gamma,\gamma}}{\rho^{(2)*}_{s,s}}\right)^{\frac{1}{2}}\cdot\rho^{(2)*}_{i,k};$$

$$(\Delta_\xi Q_{i,k\gamma})_0 = -15\cdot S_2^*\cdot\overline{u^2}\cdot[\overline{\gamma^2}]^{\frac{1}{2}} \times\left(1+2\frac{\rho^{(0)*}_{\gamma,\gamma}}{\rho^{(2)*}_{s,s}}\right)^{\frac{1}{2}}\cdot\rho^{(2)*}_{i,k},$$

where

$$S_1^* = \frac{\overline{\frac{\partial u_r^2}{\partial x_r}\cdot\frac{\partial\gamma}{\partial x_r}} + 2\overline{\frac{\partial u_n^2}{\partial x_r}\cdot\frac{\partial\gamma}{\partial x_r}}}{\left[\overline{\left(\frac{\partial u_r^2}{\partial x_r}\right)^2}\right]^{\frac{1}{2}}\cdot\left[\overline{\left(\frac{\partial\gamma}{\partial x_r}\right)^2}\right]^{\frac{1}{2}}},$$

$$S_2^* = \frac{\overline{\frac{\partial u_r}{\partial x_r}\cdot\frac{\partial u_r\gamma}{\partial x_r}}}{\left[\overline{\left(\frac{\partial u_r}{\partial x_r}\right)^2}\right]^{\frac{1}{2}}\cdot\left[\overline{\left(\frac{\partial u_r\gamma}{\partial x_r}\right)^2}\right]^{\frac{1}{2}}}$$

are dimensionless statistical coefficients characterizing isotropic velocity and scalar property fields.

By taking account of the previous relations and condition (1.18) for homogeneous anisotropic turbulence we arrive at

$$\left.\begin{aligned}(\Delta_\xi\overline{u_iu_k\gamma'})_0 &= -2(\sqrt{2})\cdot S_1\cdot\overline{q^2}\cdot[\overline{\gamma^2}]^{\frac{1}{2}} \times\left(\frac{\rho^{(0)}_{\gamma,\gamma}}{\rho^{(2)}_{s,s}}\right)^{\frac{1}{2}}\cdot\rho^{(2)}_{i,k};\\ (\Delta_\xi\overline{u_iu_k'\gamma'})_0 &= -5\cdot S_2\cdot\overline{q^2}\cdot[\overline{\gamma^2}]^{\frac{1}{2}} \times\left(1+\frac{\rho^{(0)}_{\gamma,\gamma}}{\rho^{(2)}_{s,s}}\right)^{\frac{1}{2}}\cdot\rho^{(2)}_{i,k}\end{aligned}\right\} \quad (2.13)$$

where

$$\left.\begin{aligned}S_1 &= -\frac{\sqrt{15}}{2}\cdot\frac{1}{(\overline{q^2})^{\frac{1}{2}}} \times\frac{(\Delta_\xi\overline{u_su_s\gamma'})_0}{(-\Delta_\xi\overline{u_su_s'})_0^{\frac{1}{2}}\cdot(-\Delta_\xi\overline{\gamma\gamma'})_0^{\frac{1}{2}}};\\ S_2 &= -(\sqrt{3}) \times\frac{(\Delta_\xi\overline{u_su_s'\gamma'})_0}{[3\overline{\gamma^2}(-\Delta_\xi\overline{u_su_s'})_0^2 + 5\overline{q^2}(-\Delta_\xi\overline{u_su_s'})_0(-\Delta_\xi\overline{\gamma\gamma'})_0]^{\frac{1}{2}}}\end{aligned}\right\} \quad (2.14)$$

are dimensionless statistical coefficients characterizing homogeneous anisotropic velocity and scalar property fields.

In addition, for homogeneous turbulence we may obtain the following relationships

$$\frac{1}{\rho}(\Delta_\xi\overline{\gamma p'})_0 = -\left(\frac{\partial^2}{\partial\xi_m\partial\xi_n}\overline{u_m'u_n'\gamma}\right)_0;$$

$$\frac{1}{\rho}\left(\Delta_\xi\frac{\partial}{\partial\xi_k}\overline{u_i\gamma p'}\right)_0 = -\left(\frac{\partial^3}{\partial\xi_m\partial\xi_n\partial\xi_k}\overline{u_m'u_n'u_i\gamma}\right)_0;$$

$$\left(\frac{\partial^2}{\partial\xi_m\partial\xi_j}\overline{\gamma u_iu_n'}\right)_0 = \frac{1}{3}\overline{\gamma q^2}\cdot\rho^{(2)}_{\gamma s,s}\cdot F^{in}_{mj},$$

where

$$F^{in}_{mj} = \tfrac{1}{2}(1-4\bar{c}_0)(\delta_{im}\delta_{jn}+\delta_{mn}\delta_{ij}) - 2R_{\gamma in}\cdot\delta_{jm} + \bar{c}_0(R_{\gamma im}\cdot\delta_{jn} + R_{\gamma mn}\cdot\delta_{ij} + R_{\gamma ij}\delta_{mn} + R_{\gamma jn}\cdot\delta_{in});$$

$$\rho^{(2)}_{\gamma s,s} = -\frac{1}{5\overline{\gamma q^2}}\cdot(\Delta_\xi\overline{\gamma u_su_s'})_0; \qquad R_{\gamma in} = \frac{\overline{\gamma u_iu_n}}{\frac{1}{3}\overline{\gamma q^2}}.$$

By allowing for the properties of invariance (1.19), the possibility of applying the concept of quasi-homogeneity to determination of the terms of type (1.15) included by (2.5)–(2.10) and those accounting for relationships (2.4) and (1.24), we may rewrite the system of equations (2.2), (2.3), (2.9)–(2.11) in the form

$$\frac{\partial}{\partial\tau}\overline{u_i\gamma} + U_k\frac{\partial}{\partial x_k}\overline{u_i\gamma} + \overline{u_iu_k}\frac{\partial\Gamma}{\partial x_k} + \overline{u_k\gamma}\cdot\frac{\partial U_i}{\partial x_k} + \frac{\partial}{\partial x_k}\overline{u_iu_k\gamma} + \frac{1}{2\rho}\frac{\partial}{\partial x_i}\overline{\gamma p} - \frac{1}{4}(\lambda+\gamma)\cdot\Delta_x\overline{u_i\gamma} = 0;$$

$$\frac{\partial}{\partial\tau}\overline{u_iu_k\gamma} + U_l\cdot\frac{\partial}{\partial x_l}\overline{u_iu_k\gamma} + \overline{u_iu_l\gamma}\cdot\frac{\partial U_k}{\partial x_l} + \overline{u_iu_ku_l}\times\frac{\partial\Gamma}{\partial x_l} + \overline{u_lu_k\gamma}\cdot\frac{\partial U_i}{\partial x_l} + \overline{u_ku_l}\cdot\frac{\partial}{\partial x_l}\overline{u_i\gamma} + \overline{u_l\gamma}\times\frac{\partial}{\partial x_l}\overline{u_iu_k} + \overline{u_iu_l}\cdot\frac{\partial}{\partial x_l}\overline{u_k\gamma} + \frac{1}{2\rho}\left(\frac{\partial}{\partial x_k}\overline{u_i\gamma p} + \frac{\partial}{\partial x_i}\overline{u_k\gamma p}\right) + \frac{1}{\rho}\left[\left(\frac{\partial}{\partial\xi_k}\overline{u_i\gamma p'}\right)_0 + \left(\frac{\partial}{\partial\xi_i}\overline{u_k\gamma p'}\right)_0\right] - \frac{1}{4}(2\gamma+\lambda)\,\Delta_x\overline{u_iu_k\gamma} + \frac{2}{3}\rho^{\frac{1}{2}}_{s,s}\cdot\left[R_{ij} + \frac{1}{5}(1-4\bar{c}_0)(R_{ij}-\delta_{ij})\right] \times\left[\sqrt{2}\cdot\lambda\cdot S_1\cdot\overline{q^2}^{\frac{1}{2}}\cdot\overline{\rho^{(0)}_{\gamma,\gamma}}^{\frac{1}{2}} + 5\nu \times S_2\left(\overline{\gamma^2} + 2\overline{q^2}\cdot\frac{\overline{\rho^{(0)}_{\gamma,\gamma}}}{\rho_{s,s}}\right)^{\frac{1}{2}}\cdot\rho^{\frac{1}{2}}_{s,s}\right] = 0;$$

$$\frac{1}{4\rho}\Delta_x\overline{u_i\gamma p} + \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(\frac{\partial}{\partial\xi_j}\overline{u_i\gamma p'}\right)_0 + \frac{\partial U_m}{\partial x_n}\cdot\frac{\partial}{\partial x_m}\overline{u_iu_n\gamma} + \frac{1}{4}\frac{\partial^2}{\partial x_m\partial x_n} \times(\overline{u_mu_n}\cdot\overline{u_i\gamma} + \overline{u_mu_i}\cdot\overline{u_n\gamma} + \overline{u_m\gamma}\cdot\overline{u_nu_i}) = 0;$$

$$\frac{1}{4\rho}\Delta_x\left(\frac{\partial}{\partial\xi_j}\overline{u_i\gamma p'}\right)_0 + \frac{1}{2}\cdot\frac{\partial^2 U_m}{\partial x_j\cdot\partial x_n}\cdot\frac{\partial}{\partial x_m}\overline{u_iu_n\gamma} + \frac{2}{3}\cdot S_2\cdot\left(\overline{\gamma^2} + 2\overline{q^2}\cdot\frac{\rho_{\gamma,\gamma}}{\rho_{s,s}}\right)^{\frac{1}{2}} \times\rho_{s,s}\cdot\frac{\partial U_m}{\partial x_n}\cdot F^{in}_{mj} = 0$$

where

$$\overline{\rho^{(0)}_{\gamma,\gamma}} = \overline{\gamma^2}\cdot\rho^{(0)}_{\gamma,\gamma}.$$

The above system is determined (except for two statistical coefficients $S_1$ and $S_2$), if the problem of distribution of the values $\gamma^2$ and $\rho^{(0)}_{\gamma,\gamma}$ in the field of non-homogeneous turbulence is solved. This problem will be discussed in the next section.

The numerical values of the statistical coefficients $S_1^*$ and $S_2^*$ may be estimated from the inequality (1.30)

$$|S_1^*| \leqslant 1 + 2\sqrt{2}; \qquad |S_2^*| \leqslant 1.$$

These inequalities may, probably, be used for rough estimation of the coefficients (2.14).

### 3. THE FIELD OF SCALAR PROPERTY FLUCTUATIONS

The equations describing the field of scalar property fluctuations at non-homogeneous turbulence are as follows:

(i) equations of double one-point correlation [14]

$$\frac{\partial}{\partial\tau}\overline{\gamma^2} + U_k\frac{\partial}{\partial x_k} + 2\overline{u_k\gamma}\frac{\partial\Gamma}{\partial x_k} + \frac{\partial}{\partial x_k}\overline{u_k\gamma^2} - \lambda\Delta\overline{\gamma^2} + 2\lambda\cdot\overline{\left(\frac{\partial\gamma}{\partial x_k}\right)^2} = 0; \tag{3.1}$$

(ii) equation of triple correlations

$$\frac{\partial}{\partial\tau}\overline{u_k\gamma^2} + U_l\frac{\partial}{\partial x_l}\overline{u_k\gamma^2} + \overline{u_l\gamma^2}\frac{\partial U_k}{\partial x_l} + 2\overline{u_ku_l\gamma}\cdot\frac{\partial\Gamma}{\partial x_l} + \frac{\partial}{\partial x_l}\overline{u_ku_l\gamma^2} - \overline{\gamma^2}\times\frac{\partial}{\partial x_l}\overline{u_ku_l} - 2\overline{u_k\gamma}\cdot\frac{\partial}{\partial x_l}\overline{u_l\gamma} + \frac{1}{\rho}\overline{\gamma^2\frac{\partial}{\partial x_k}} - \overline{\nu\gamma^2\cdot\frac{\partial^2 u_k}{\partial x_l^2}} - 2\lambda\cdot\overline{u_k\gamma\cdot\frac{\partial^2\gamma}{\partial x_l^2}} = 0, \tag{3.2}$$

where the correlation $\overline{u_ku_l\gamma}$ is determined by equation (2.3).

By using Millionshchikov's hypothesis

$$\overline{u_ku_l\gamma^2} = \overline{u_ku_l}\cdot\overline{\gamma^2} + 2\overline{u_k\gamma}\cdot\overline{u_l\gamma} \tag{3.3}$$

the set of equations for correlations is limited to equation (3.2). As before (see Sec. 1 and 2), the unknown terms in (3.1) and (3.2) are written in the new system of coordinates (1.6)

$$2\lambda\overline{\left(\frac{\partial\gamma}{\partial x_k}\right)^2} = \frac{1}{2}\lambda\cdot\Delta_x\overline{\gamma^2} + 12\cdot\lambda\cdot\overline{\rho^{(0)}_{\gamma,\gamma}}; \quad (3.4)$$

$$\overline{\nu\gamma^2\frac{\partial^2 u_k}{\partial x_l^2}} + 2\lambda\cdot\overline{u_k\gamma\frac{\partial^2\gamma}{\partial x_l^2}} = \frac{1}{4}(2\lambda+\nu)\Delta_x\overline{u_k\gamma^2} + \nu(\Delta_\xi\overline{u_k\gamma'\gamma'})_0 - \nu\frac{\partial}{\partial x_l}\left(\frac{\partial}{\partial\xi_l}\overline{u_k\gamma'\gamma'}\right)_0 + 2\lambda\cdot(\Delta_\xi\overline{\gamma\gamma' u'_k})_0 + 2\lambda\cdot\frac{\partial}{\partial x_l}\left(\frac{\partial}{\partial\xi_l}\overline{\gamma\gamma' u'_k}\right)_0; \quad (3.5)$$

$$\overline{\gamma^2\frac{\partial p}{\partial x_k}} = \frac{1}{2}\frac{\partial}{\partial x_k}\overline{\gamma^2 p} + \left(\frac{\partial}{\partial\xi_k}\overline{\gamma^2 p'}\right)_0. \quad (3.6)$$

Equation for one-point correlation $\overline{\gamma^2 p}$ included in (3.6) is derived from (1.11) and presented in the form

$$\frac{1}{\rho}\left[\frac{1}{4}\Delta_x\overline{p\gamma^2} + (\Delta_\xi\overline{\gamma^2 p'})_0 + \frac{\partial}{\partial x_k}\left(\frac{\partial}{\partial\xi_k}\overline{\gamma^2 p'}\right)_0\right] = -2\frac{\partial U_m}{\partial x_n}\left[\frac{1}{2}\frac{\partial}{\partial x_m}\overline{u_n\gamma^2} + \left(\frac{\partial}{\partial\xi_m}\overline{\gamma^2 u'_n}\right)_0\right] - \frac{1}{4}\cdot\frac{\partial^2}{\partial x_m\partial x_n}\overline{u_m u_n\gamma^2} - \frac{1}{2}\left[\frac{\partial}{\partial x_m}\times\left(\frac{\partial}{\partial\xi_n}\overline{u'_m u'_n\gamma^2}\right)_0 + \frac{\partial}{\partial x_n}\left(\frac{\partial}{\partial\xi_m}\overline{u'_m u'_n\gamma^2}\right)_0\right] - \left(\frac{\partial^2}{\partial\xi_m\partial\xi_n}\overline{u'_m u'_n\gamma^2}\right)_0. \quad (3.7)$$

In order to derive a differential equation describing the change in the field of non-homogeneous turbulence of scalar substance microscale, the dynamic equation of double scalar substance correlation is used [15] which, while presented in the oordinate system of (1.6), is of the form

$$\frac{\partial}{\partial\tau}\overline{\gamma\gamma'} + \overline{u_k\gamma'}\left(\frac{\partial\Gamma}{\partial x_k}\right)_A + \overline{u'_k\gamma}\left(\frac{\partial\Gamma}{\partial x_k}\right)_B + \frac{1}{2}[(U_k)_A + (U_k)_B]\cdot\left(\frac{\partial}{\partial x_k}\right)_{AB}\overline{\gamma\gamma'} + [(U_k)_B - (U_k)_A]\cdot\frac{\partial}{\partial\xi_k}\overline{\gamma\gamma'} + \frac{1}{2}\left(\frac{\partial}{\partial x_k}\right)_{AB}\times(\overline{u_k\gamma\gamma'} - \overline{u'_k\gamma'\gamma}) + \frac{\partial}{\partial\xi_k}(\overline{u'_k\gamma'\gamma} - \overline{u_k\gamma\gamma'}) - \frac{1}{2}\cdot\lambda\cdot(\Delta_x)_{AB}\overline{\gamma\gamma'} - 2\cdot\lambda\cdot\Delta_\xi\overline{\gamma\gamma'} = 0. \quad (3.8)$$

The operation upon (3.8) with the operator $\partial^2/\partial\xi_s\partial\xi_p$ and assumption of $\vec{\xi} = 0$ yields

$$\frac{\partial}{\partial\tau}\left(\frac{\partial^2}{\partial\xi_s\partial\xi_p}\overline{\gamma\gamma'}\right)_0 + U_k\frac{\partial}{\partial x_k}\left(\frac{\partial^2}{\partial\xi_s\partial\xi_p}\overline{\gamma\gamma'}\right)_0 + \left(\frac{\partial^2}{\partial\xi_k\partial\xi_p}\overline{\gamma\gamma'}\right)_0\frac{\partial U_k}{\partial x_s} + \left(\frac{\partial^2}{\partial\xi_k\partial\xi_s}\overline{\gamma\gamma'}\right)_0\times\frac{\partial U_k}{\partial x_p} + \frac{1}{4}\cdot\frac{\partial^2 U_k}{\partial x_s\partial x_p}\cdot\frac{\partial}{\partial x_k}\overline{\gamma^2} + \frac{1}{2}\overline{u_k\gamma}\cdot\frac{\partial^3\Gamma}{\partial x_p\partial x_s\partial x_k} - \frac{\partial^2\Gamma}{\partial x_p\partial x_k}\cdot\left(\frac{\partial}{\partial\xi_s}\overline{u_k\gamma'}\right)_0 - \frac{\partial^2\Gamma}{\partial x_s\partial x_k}\cdot\left(\frac{\partial}{\partial\xi_p}\overline{u_k\gamma'}\right)_0 + \frac{1}{2}\frac{\partial}{\partial x_k}\left[\frac{\partial^2}{\partial\xi_s\partial\xi_p}(\overline{u_k\gamma\gamma'} + \overline{u'_k\gamma'\gamma})\right]_0 + \left(\frac{\partial^3}{\partial\xi_k\partial\xi_s\partial\xi_p}\overline{u'_k\gamma'\gamma}\right)_0 - \left(\frac{\partial^3}{\partial\xi_k\partial\xi_s\partial\xi_p}\overline{u_k\gamma\gamma'}\right)_0 - \frac{1}{2}\cdot\lambda\cdot\Delta_x\left(\frac{\partial^2}{\partial\xi_s\partial\xi_p}\overline{\gamma\gamma'}\right)_0 - 2\lambda\cdot\left(\Delta_\xi\frac{\partial^2}{\partial\xi_s\partial\xi_p}\overline{\gamma\gamma'}\right)_0 = 0. \quad (3.9)$$

Similarly to (1.28) and (2.13) it may be shown that for homogeneity the following relationships hold

$$\left(\Delta_\xi\frac{\partial}{\partial\xi_k}\overline{u_k\gamma\gamma'}\right)_0 = \frac{15}{\sqrt{3}}S_\gamma\cdot\overline{\rho^{(0)}_{\gamma,\gamma}}\cdot\overline{\rho_{s,s}}^{\frac{1}{2}};$$

$$(\Delta_\xi\Delta_\xi\overline{\gamma\gamma'})_0 = \frac{10}{\sqrt{3}}\cdot S_\lambda\cdot\frac{1}{\lambda}\cdot\overline{\rho^{(0)}_{\gamma,\gamma}}\cdot\overline{[\rho_{s,s}]}^{\frac{1}{2}}, \quad (3.10)$$

where $S_\gamma$ and $S_\lambda$ are the dimensionless statistical coefficients characterizing homogeneous anisotropic scalar property field. They can be approximated by the coefficients†

$$S_\gamma^* = \frac{\overline{\left(\frac{\partial\gamma}{\partial x_r}\right)^2 \frac{\partial u_r}{\partial x_r}}}{\overline{\left(\frac{\partial\gamma}{\partial x_r}\right)^2} \cdot \left[\overline{\left(\frac{\partial u_r}{\partial x_r}\right)^2}\right]^{\frac{1}{2}}};$$

and

$$S_\lambda^* = \lambda \frac{\overline{\left(\frac{\partial^2\gamma}{\partial x_r^2}\right)^2}}{\overline{\left(\frac{\partial\gamma}{\partial x_r}\right)^2} \cdot \left[\overline{\left(\frac{\partial u_r}{\partial x_r}\right)^2}\right]^{\frac{1}{2}}},$$

which are statistical characteristics of isotropic scalar property field.

Taking into account the properties of invariance (1.19) and relationships (2.11), (3.3)–(3.6) and (3.10), the set of equations (3.1), (3.2), (3.7) and (3.9) is written in the form

$$\frac{\partial}{\partial\tau}\overline{\gamma^2} + U_k \frac{\partial}{\partial x_k}\overline{\gamma^2} + 2\overline{u_k\gamma}\frac{\partial\Gamma}{\partial x_k}\overline{u_k\gamma^2} - \tfrac{1}{2}.\lambda.\Delta_x\overline{\gamma^2} + 12\lambda.\overline{\rho_{\gamma,\gamma}^{(0)}} = 0;$$

$$+\frac{\partial}{\partial x_k}\frac{\partial}{\partial\tau}\overline{u_k\gamma^2} + U_l\frac{\partial}{\partial x_l}\overline{u_k\gamma^2} + \overline{u_l\gamma^2}.\frac{\partial U_k}{\partial x_l} + \overline{u_ku_l\gamma}\frac{\partial\Gamma}{\partial x_l} + \overline{u_ku_l}.\frac{\partial\overline{\gamma^2}}{\partial x_l} + 2\overline{u_l\gamma}.\frac{\partial}{\partial x_l}\overline{u_k\gamma} + \frac{1}{2\rho}.\frac{\partial}{\partial x_k}\overline{\gamma^2 p} - \tfrac{1}{4}(2\lambda+\nu)\Delta_x\overline{u_k\gamma^2} = 0;$$

$$\frac{1}{4\rho}\Delta_x\overline{\gamma^2 p} + \frac{\partial U_m}{\partial x_n}.\frac{\partial}{\partial x_m}\overline{u_n\gamma^2} - \frac{1}{4}.\frac{\partial^2}{\partial x_m\partial x_n} \times (\overline{u_mu_n}.\overline{\gamma^2} + 2\overline{u_m\gamma}.\overline{u_n\gamma}) = 0;$$

$$\frac{\partial}{\partial\tau}\overline{\rho_{s,p}^{(0)}} + U_k.\frac{\partial}{\partial x_k}\overline{\rho_{s,p}^{(0)}} + \frac{\partial U_k}{\partial x_s}.\overline{\rho_{k,p}^{(0)}} + \frac{\partial U_k}{\partial x_p}.\overline{\rho_{k,s}^{(0)}} - \frac{1}{24}.\frac{\partial^2 U_k}{\partial x_p\partial x_s}.\frac{\partial}{\partial x_k}\overline{\gamma^2} - \frac{1}{12}\overline{u_k\gamma}.\frac{\partial^3\Gamma}{\partial x_k\partial x_p\partial x_s} + \frac{5}{\sqrt{3}}.\rho_{s\,p}^{(0)}.[\overline{\rho}_{s,s}]^{\frac{1}{2}}.(S_\gamma + 2S_\lambda) - \frac{1}{2}.\lambda.\Delta_x\rho_{s,p}^{(0)} = 0.$$

† The first of the coefficients was discussed first in [15].

The above system of equations describing the field of scalar property fluctuations at non-homogeneous turbulence is determined except for the two universal statistical coefficients $S_\gamma$ and $S_\lambda$ (the problems of momentum and scalar property transfer are solved in Sec. 1 and 2). The numerical values of these coefficients may be estimated for isotropic turbulence by the Betchov inequality for fixed moments of velocity fluctuations and sclar property derivatives [15] and equations for the value $(\Delta_\xi Q_{\gamma,\gamma})_0$ using the first Kolmogoroff similarity hypothesis. The estimations of $S_\gamma^*$ and $S_\lambda^*$ are as follows

$$|S_\gamma^*| \leqslant \tfrac{2}{3}(\delta_\gamma^*)^{\frac{1}{2}} \simeq \frac{2}{\sqrt{3}}; \qquad S_\lambda^* = -\tfrac{3}{2}S_\gamma^*,$$

where $\delta_\gamma^*$ is the flatness factor of the probability density distribution of scalar property derivatives.

## REFERENCES


1. M. D. MILLIONSHCHIKOV, On theory of homogeneous isotropic turbulence, *Dokl. Akad. Nauk SSSR* **32**, 9 (1941).
2. J. LAUFER, Structure of turbulence in fully developed pipe flow, Nat. Adv. Com. Aeronaut., Rep. No. 1174 (1954).
3. P. S. KLEBANOFF, Characteristics of turbulence in a boundary layer with zero pressure gradient, Nat. Adv. Com. Aeronaut., Rep. No. 1247 (1955).
4. G. COMPTE-BELLOT, Ecoulement turbulent entre deux parois paralleles, Paris, Publication Scientifiques et Techniques du Minister de L'air (1965).
5. J. O. HINZE, *Turbulence*. McGraw-Hill, New York (1959).
6. A. S. MONIN and A. M. YAGLOM, *Statistical Hydromechanics*, part I. Nauka, Moscow (1965).
7. P. Y. CHOU, On velocity correlations and the solutions of the equations of turbulent fluctuation, *Q. J. Appl. Math.* **3**, 38–54 (1945).
8. H. P. ROBERTSON, The invariant theory of isotropy, *Proc. Camb. Phil. Soc.* **36**, 2 (1940).
9. A. S. MONIN and A. M. YAGLOM, *Statistical Hydromechanics*, part II. Nauka, Moscow (1967).
10. M. S. UBEROI, Energy transfer in isotropic turbulence, *J. Fluid Mech.* **6**, 8 (1963).
11. R. BECTCHOV, Inequality concerning the production of vorticity in isotropic turbulence, *J. Fluid Mech.* **1**, 5 (1956).

12. B. A. KOLOVANDIN, On calculation of basic heat transfer characteristics for turbulent shear flows, *Heat and Mass Transfer*, Vol. I, pp. 154–166. Energia, Moscow (1968).
13. S. CORRSIN, Heat transfer in isotropic turbulence, *J. Appl. Phys.* **23**, 1 (1952).
14. A. M. YAGLOM, Local temperature field structure in a turbulent flow, *Dokl. Akad. Nauk SSSR* **69**, 6 (1949).
15. A. M. YAGLOM, Inequality for a mixed moment of velocity and temperature derivatives in locally isotropic turbulence, *Izv. Akad. Nauk SSSR, ser. Fiz. Atmosf. i Okeana* **3**, 9 (1967).

## THEORIE STATISTIQUE DE TRANSFERT DANS UNE TURBULENCE NON HOMOGENE

**Résumé**—On considère l'un des modèles possibles de description statistique du transfert de quantité de mouvement et de propriété scalaire (température, concentration du mélange) dans un écoulement turbulent incompressible et non homogène. Le modèle est basé sur l'utilisation d'équations en nombre fini de corrélations. Afin de déterminer ces équations, on a utilisé quelques expressions approchées pour des corrélations anisotropes entre deux points et des lois caractérisant le transfert de quantité de mouvement et de propriété scalaire dans une turbulence homogène et anisotrope.

## STATISTISCHE ÜBERTRAGUNGSTHEORIE IN NICHTHOMOGENER TURBULENZ

**Zusammenfassung**—Es wird eines der möglichen Modelle der statistischen Beschreibung von Impulsübertragung und der Übertragung von skalaren Eigenschaften (Temperatur, Mischkonzentration) in einer inhomogenen, turbulenten, inkompressiblen Strömung betrachtet. Das Modell benützt eine endliche Anzahl von Ein-Punkt Beziehungsgleichungen. Um die Gleichungen zu bestimmen, wurden einige Näherungsausdrücke für anisotrope Zwei-Punkt-Gleichungen verwendet, die den Impulsaustausch und die Übertragung skalarer Eigenschaften in homogener und anisotroper Turbulenz charakterisieren.

## СТАТИСТИЧЕСКАЯ ТЕОРИЯ ПЕРЕНОСА В НЕОДНОРОДНОЙ ТУРБУЛЕНТНОСТИ

**Аннотация**—Рассматривается одна из возможных моделей статистического описания переноса импульса и скалярной субстанции (температуры, концентрации пассивной примоси) в неоднородном турбулентном несжимаемом потоке. Модель основывается на использовании конечного числа уравнений для одноточечных корреляций. Для замыкания уравнений используются некоторые приближенные выражения для неизотропных двухточечных корреляций и соотношения, характеризующие перенос импульса и скалярной субтанции в однородной неизотропной турбулентности.